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AZUSA GDOPS PROGRAM
AZGD2

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TABLE OF CONTENTS

SECTION I
INTRODUCTION

The AZUSA Geometric Dilution of Precision (GDOPS) Program (AZGD2) is designed to compute GDOPS for a standard KSC theoretical trajectory based on an AZUSA (or AZUSA-like) system. Internally the AZGD2 program uses a right-handed cartesian coordinate system located at the origin (φ_0, λ_0, h_0) with X downrange at some azimuth from north, Z crossrange 90 degrees greater than the azimuth of X, and Y perpendicular to the XZ plane. The output may be in this coordinate system or in the so called Apollo Saturn Coordinate System, which differs from the aforementioned system only in that its Z is downrange, its Y is crossrange, and its X is perpendicular, i.e., the axes are relabeled.

SECTION II
DEFINITION OF SYMBOLS AND TERMS

Symbol	Definition
a	Semimajor axis of Earth ellipsoid (input)
b	Semiminor axis of Earth ellipsoid (input)
AZ	Azimuth of X-axis (input)
e^2	Square of ellipsoid eccentricity
φ, λ, h	Geographic location (latitude, longitude, altitude) subscripts: o : origin, J : baselines, J = 1,5 (see below for identification) (input)
$\sigma_\ell, \sigma_{\dot{\ell}}$	Sigma for L-cosine, sigma for L-cosine rate (input)
$\sigma_m, \sigma_{\dot{m}}$	Sigma for M-cosine, sigma for M-cosine rate (input)
$\sigma_R, \sigma_{\dot{R}}$	Sigma for range, sigma for range rate (input)
X,Y,Z	Vehicle cartesian position (input)
$\dot{X}, \dot{Y}, \dot{Z}$	Vehicle cartesian velocity (input)
$\sigma_X, \sigma_{\dot{X}}$	Expected X-position and X-velocity errors
$\sigma_Y, \sigma_{\dot{Y}}$	Expected Y-position and Y-velocity errors
$\sigma_Z, \sigma_{\dot{Z}}$	Expected Z-position and Z-velocity errors
σ_V	Expected vector resultant velocity error
X_J, Y_J, Z_J	Cartesian location of baseline points

Symbol	Definition
$\underline{J}$	
1, 2	L baseline end points
3, 4	M baseline end points
5	Range tracking point

SECTION III
COMPUTER PROGRAM EQUATIONS AND LOGIC

A. GEOXYZ

Geographic to cartesian position subroutine:

$$N_o = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi_o}}$$

$$U_o = (N_o + h_o) \cos \varphi_o$$

$$V_o = [(1 - e^2) N_o + h_o] \sin \varphi_o$$

$$W_o = 0$$

$$N_J = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi_J}}$$

$$U_J = (N_J + h_J) \cos \varphi_J \cos (\lambda_o - \lambda_J)$$

$$V_J = [(1 - e^2) N_J + h_J] \sin \varphi_J$$

$$W_J = (N_J + h_J) \cos \varphi_J \sin (\lambda_o - \lambda_J)$$

$$U = U_J - U_o$$

$$V = V_J - V_o$$

$$W = W_J - W_o$$

$$X_J = -U \cos AZ \cos \varphi_0 + V \cos AZ \sin \varphi_0 - W \sin AZ$$

$$Y_J = U \cos \varphi_0 + V \sin \varphi_0$$

$$Z_J = U \sin AZ \sin \varphi_0 - V \cos \varphi_0 \sin AZ - W \cos AZ$$

B. MAIN PROGRAM

$$XLL = \frac{X_1 + X_2}{2}$$

$$YLL = \frac{Y_1 + Y_2}{2}$$

$$ZLL = \frac{Z_1 + Z_2}{2}$$

$$XMM = \frac{X_3 + X_4}{2}$$

$$YMM = \frac{Y_3 + Y_4}{2}$$

$$ZMM = \frac{Z_3 + Z_4}{2}$$

$$XLX = X_2 - X_1$$

$$XLY = Y_2 - Y_1$$

$$XLZ = Z_2 - Z_1$$

$$XMX = X_4 - X_3$$

$$XMY = Y_4 - Y_3$$

$$XMZ = Z_4 - Z_3$$

$$B_L = (XLX^2 + YLY^2 + ZLZ^2)^{1/2}$$

$$B_M = (XMX^2 + XMY^2 + XMZ^2)^{1/2}$$

$$CXL = XLX/B_L$$

$$CYL = YLY/B_L$$

$$CZL = ZLZ/B_L$$

$$CXM = XMX/B_M$$

$$CYM = YMY/B_M$$

$$CZM = ZMZ/B_M$$

$$XXL = X - XLL$$

$$YYL = Y - YLL$$

$$ZZL = Z - ZLL$$

$$XXM = X - XMM$$

$$YYM = Y - YMM$$

$$ZZM = Z - ZMM$$

$$XXR = X - X_5$$

$$YYR = Y - X_5$$

$$ZZR = Z - X_5$$

C. DLMXYZ

Partial derivative computation subroutine:

(A = XXL, XXM; B = YYL, YYM; C = ZZL, ZZM; D = CXL, CXM;
E = CYL, CYM; F = CZL, CZM; input arguments)

$W_1^{(i)}, W_2^{(i)}, W_3^{(i)}$ are elements of a row vector. (See PHIGAM, paragraph E.)

$$P = (A^2 + B^2 + C^2)^{1/2}$$

$$CVX = A/P$$

$$CVY = B/P$$

$$CVZ = C/P$$

$$W_1^{(i)} = (1 - CVX^2)D/P - CVX \cdot CVY \cdot E/P - CVX \cdot CVZ \cdot F/P$$

$$W_2^{(i)} = -CVX \cdot CVY \cdot D/P + (1 - CVY^2)E/P - CVY \cdot CVZ \cdot F/P$$

$$W_3^{(i)} = -CVX \cdot CVZ \cdot D/P - CVY \cdot CVZ \cdot E/P + (1 - CVZ^2) \cdot F/P$$

$$i = L, M$$

D. DRXYZ

Partial derivative computation subroutine:

$$R = (XXR^2 + YYR^2 + ZZR^2)^{1/2}$$

$$W_1^{(R)} = XXR/R$$

$$W_2^{(R)} = YYR/R$$

$$W_3^{(R)} = ZZR/R$$

($W_1^{(R)}$, $W_2^{(R)}$, $W_3^{(R)}$, are elements of a row vector.)

E. PHIGAM

Partial derivative matrix subroutine:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} W_1^{(L)} & W_2^{(L)} & W_3^{(L)} \\ W_1^{(M)} & W_2^{(M)} & W_3^{(M)} \\ W_1^{(R)} & W_2^{(R)} & W_3^{(R)} \end{bmatrix}$$

F. VARCOV

Variance-covariance matrix, and sigma computation subroutine:

($S_1 = \sigma_\ell, \sigma_{\dot{\ell}}$; $S_2 = \sigma_m, \sigma_{\dot{m}}$; $S_3 = \sigma_R, \sigma_{\dot{R}}$; input arguments. Output arguments $SX = \sigma_X, \sigma_{\dot{X}}$; $SY = \sigma_Y, \sigma_{\dot{Y}}$; $SZ = \sigma_Z, \sigma_{\dot{Z}}$)

$$AL = p_{11}/S_1; \quad BL = p_{12}/S_1; \quad CL = p_{13}/S_1$$

$$AM = p_{21}/S_2; \quad BM = p_{22}/S_2; \quad CM = p_{23}/S_2$$

$$AR = p_{31}/S_3; \quad BR = p_{32}/S_3; \quad CR = p_{33}/S_3$$

$$AA = AL^2 + AM^2 + AR^2$$

$$AB = AL \cdot BL + AM \cdot BM + AR \cdot BR$$

$$AC = AL \cdot CL + AM \cdot CM + AR \cdot CR$$

$$BB = BL^2 + BM^2 + BR^2$$

$$BC = BL \cdot CL + BM \cdot CM + BR \cdot CR$$

$$CC = CL^2 + CM^2 + CR^2$$

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \begin{bmatrix} AA & AB & AC \\ AB & BB & BC \\ AC & BC & CC \end{bmatrix}$$

$$H^{(1)} = H^{-1}, \quad SX = \sqrt{h_{11}^{(1)}}, \quad SY = \sqrt{h_{22}^{(1)}}, \quad SZ = \sqrt{h_{33}^{(1)}}$$

(Matrix inversion accomplished by MINVRT subroutine, paragraph H.)

G. SIGVEL

Vector velocity sigma subroutine:

$$V = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}$$

$$DA = \dot{X}^2 \cdot h_{11}^{(1)}$$

$$DAB = 2\dot{X}\cdot\dot{Y}\cdot h_{12}^{(1)}$$

$$DAC = 2\dot{X}\cdot\dot{Z}\cdot h_{13}^{(1)}$$

$$DB = \dot{Y}^2 h_{22}^{(1)}$$

$$DBC = 2\dot{Y}\cdot\dot{Z}\cdot h_{23}^{(1)}$$

$$DC = \dot{Z}^2 h_{33}^{(1)}$$

$$\sigma_V = (DA + DAB + DAC + DB + DBC + DC)^{1/2}/V$$

H. MINVRT

Matrix inversion subroutine:

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$$A_1 = h_{11}(h_{22}h_{33} - h_{23}h_{32})$$

$$A_2 = h_{12}(h_{21}h_{33} - h_{23}h_{31})$$

$$A_3 = h_{13}(h_{21}h_{32} - h_{22}h_{31})$$

$$|H| = A_1 - A_2 + A_3$$

(If $|H| = 0$, matrix is singular and inversion is impossible.)

$H^{(C)} =$

$$\begin{bmatrix} \left[h_{11}^{(C)} = A_1/h_{11}\right] & \left[h_{12}^{(C)} = -(h_{12}h_{33} = h_{32}h_{13})\right] & \left[h_{13}^{(C)} = h_{12}h_{23} - h_{22}h_{13}\right] \\ \left[h_{21}^{(C)} = -A_1/h_{11}\right] & \left[h_{22}^{(C)} = h_{11}h_{33} - h_{31}h_{13}\right] & \left[h_{23}^{(C)} = -(h_{11}h_{23} - h_{21}h_{13})\right] \\ \left[h_{31}^{(C)} = A_3/h_{13}\right] & \left[h_{32}^{(C)} = -(h_{11}h_{32} - h_{12}h_{31})\right] & \left[h_{33}^{(C)} = h_{11}h_{22} - h_{21}h_{12}\right] \end{bmatrix}$$

$$H^{-1} = H^{(C)}/\,|H|$$

SECTION IV
MATHEMATICAL FORMULATION

A. SYSTEM GEOMETRY

Let $\vec{i}, \vec{j}, \vec{k}$ be the unit base vectors at the origin of a cartesian reference frame XYZ and let $\vec{B}_L$, $\vec{B}_M$ be an intersecting pair of baselines in this reference frame, viz the AZUSA baseline system. Let $\vec{R}_1$, $\vec{R}_2$ extend from the origin to either end of $\vec{B}_L$ and let $\vec{R}_3$, $\vec{R}_4$ extend similarly to either end of $\vec{B}_M$. Then, from figure 1,

$$\vec{B}_L = \vec{R}_2 - \vec{R}_1, \quad \vec{B}_M = \vec{R}_4 - \vec{R}_3 \qquad (1)$$

Then $\vec{B}_L$, $\vec{B}_M$ have direction cosines, with respect to X, Y, Z, given by:

$$\begin{aligned}
\cos(\vec{B}_L, \vec{X}) &= \vec{B}_L \cdot \vec{i} / \left|\vec{B}_L\right| \cdot \left|\vec{i}\right| & \cos(\vec{B}_M, \vec{X}) &= \vec{B}_M \cdot \vec{i} / \left|\vec{B}_M\right| \cdot \left|\vec{i}\right| \\
\cos(\vec{B}_L, \vec{Y}) &= \vec{B}_L \cdot \vec{j} / \left|\vec{B}_L\right| \cdot \left|\vec{j}\right| & \cos(\vec{B}_M, \vec{Y}) &= \vec{B}_M \cdot \vec{j} / \left|\vec{B}_M\right| \cdot \left|\vec{j}\right| \\
\cos(\vec{B}_L, \vec{Z}) &= \vec{B}_L \cdot \vec{k} / \left|\vec{B}_L\right| \cdot \left|\vec{k}\right| & \cos(\vec{B}_M, \vec{Z}) &= \vec{B}_M \cdot \vec{k} / \left|\vec{B}_M\right| \cdot \left|\vec{k}\right|
\end{aligned} \qquad (2)$$

If $\vec{B}_L = B_L^{(x)}\vec{i} + B_L^{(y)}\vec{j} + B_L^{(z)}\vec{k}$, $\vec{B}_M = B_M^{(x)}\vec{i} + B_M^{(y)}\vec{j} + B_M^{(z)}\vec{k}$

then,

$$\begin{aligned}
\vec{B}_L \cdot \vec{i} &= B_L^{(x)}\,\vec{i}\cdot\vec{i} + B_L^{(y)}\,\vec{j}\cdot\vec{i} + B_L^{(z)}\,\vec{k}\cdot\vec{i} = B_L^{(x)}, & \vec{B}_M \cdot \vec{i} &= B_M^{(x)} \\
\vec{B}_L \cdot \vec{j} &= B_L^{(y)} & \vec{B}_M \cdot \vec{j} &= B_M^{(y)} \\
\vec{B}_L \cdot \vec{k} &= B_L^{(z)} & \vec{B}_M \cdot \vec{k} &= B_M^{(z)}
\end{aligned} \qquad (3)$$

Since $|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$, it follows from equations (2) and (3) that

$$\cos(\vec{B}_L, \vec{X}) = \frac{B_L^{(x)}}{|\vec{B}_L|} \qquad \cos(\vec{B}_M, \vec{X}) = \frac{B_M^{(x)}}{|\vec{B}_M|} \tag{4}$$

$$\cos(\vec{B}_L, \vec{Y}) = \frac{B_L^{(y)}}{|\vec{B}_L|} \qquad \cos(\vec{B}_M, \vec{Y}) = \frac{B_M^{(y)}}{|\vec{B}_M|}$$

$$\cos(\vec{B}_L, \vec{Z}) = \frac{B_L^{(z)}}{|\vec{B}_L|} \qquad \cos(\vec{B}_M, \vec{Z}) = \frac{B_M^{(z)}}{|\vec{B}_M|}$$

If

$$\vec{R}_1 = X_1\vec{i} + Y_1\vec{j} + Z_1\vec{k}$$

$$\vec{R}_2 = X_2\vec{i} + Y_2\vec{j} + Z_2\vec{k}$$

$$\vec{R}_3 = X_3\vec{i} + Y_3\vec{j} + Z_3\vec{k}$$

$$\vec{R}_4 = X_4\vec{i} + Y_4\vec{j} + Z_4\vec{k}$$

then by equation (1),

$$\cos(\vec{B}_L, \vec{X}) = \frac{X_2 - X_1}{B_L} \qquad \cos(\vec{B}_M, \vec{X}) = \frac{X_4 - X_3}{B_M} \tag{5}$$

$$\cos(\vec{B}_L, \vec{Y}) = \frac{Y_2 - Y_1}{B_L} \qquad \cos(\vec{B}_M, \vec{Y}) = \frac{Y_4 - Y_3}{B_M}$$

$$\cos(\vec{B}_L, \vec{Z}) = \frac{Z_2 - Z_1}{B_L} \qquad \cos(\vec{B}_M, \vec{Z}) = \frac{Z_4 - Z_3}{B_M}$$

where:

$$B_L = B_L = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$$

$$B_M = B_M = \sqrt{(X_4 - X_3)^2 + (Y_4 - Y_3)^2 + (Z_4 - Z_3)^2}$$

Thus, the orientations of $\vec{B}_L$, $\vec{B}_M$ with respect to X, Y, Z have been obtained.

Now let $\vec{R}_L$, $\vec{R}_M$ extend from the origin to the centers of $\vec{B}_L$, $\vec{B}_M$ respectively. Then, from figure 2,

$$\vec{R}_L = \frac{\vec{B}_L}{2} + \vec{R}_1, \quad \vec{R}_M = \frac{\vec{B}_M}{2} + \vec{R}_3 \quad . \tag{6}$$

But, substituting from equation (1)

$$\vec{R}_L = \frac{\vec{R}_2 - \vec{R}_1}{2} + \vec{R}_1 = \frac{\vec{R}_2 + \vec{R}_1}{2} \tag{7}$$

$$\vec{R}_M = \frac{\vec{R}_4 - \vec{R}_3}{2} + \vec{R}_3 = \frac{\vec{R}_4 + \vec{R}_3}{2} \quad .$$

Let $\vec{R}$ be the slant range from the origin to the vehicle.

Let $\vec{L}$, $\vec{M}$ be the slant ranges from the vehicle to the centers of $\vec{B}_L$, $\vec{B}_M$ respectively. From figure 3,

$$\vec{L} = \vec{R} - \vec{R}_L, \quad \vec{M} = \vec{R} - \vec{R}_M \quad . \tag{8}$$

Following the reasoning of equations (2), (3), and (4), the direction cosines of $\vec{L}$, $\vec{M}$ with respect to X, Y, Z are given by:

$$\cos(\vec{L}, \vec{X}) = \vec{L} \cdot \frac{\vec{i}}{\vec{L} \cdot \vec{i}} = \frac{\vec{L}_x}{\vec{L}} \tag{9}$$

$$\cos(\vec{L}, \vec{Y}) = \frac{\vec{L}_y}{\vec{L}}$$

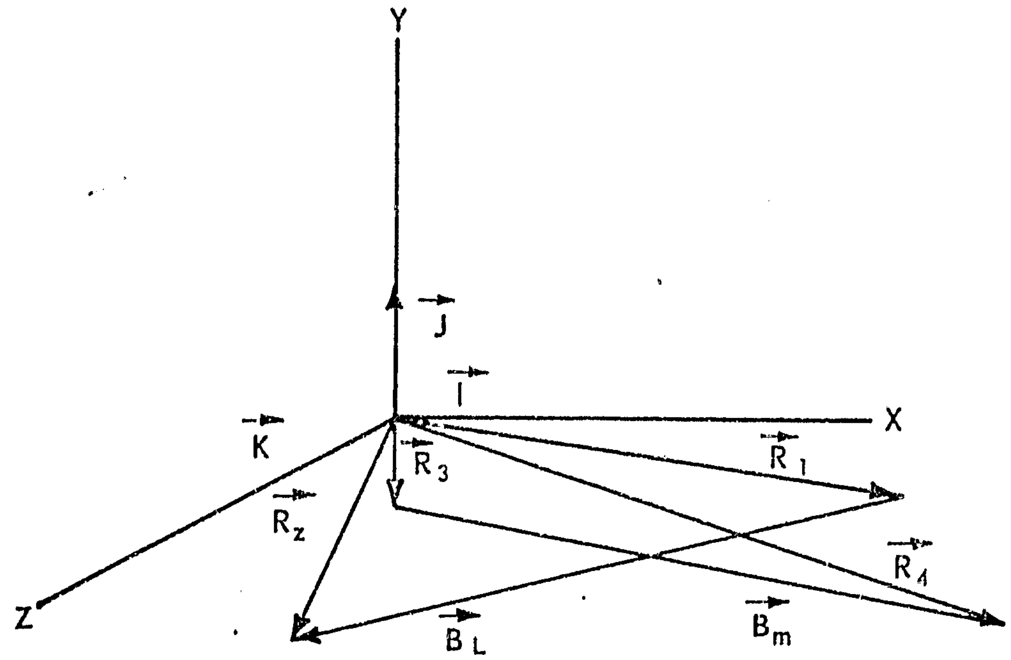


Figure 1

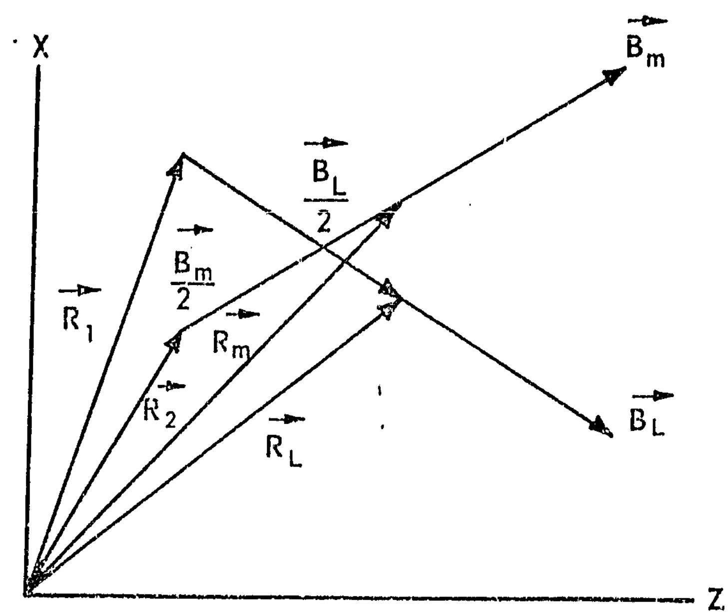


Figure 2

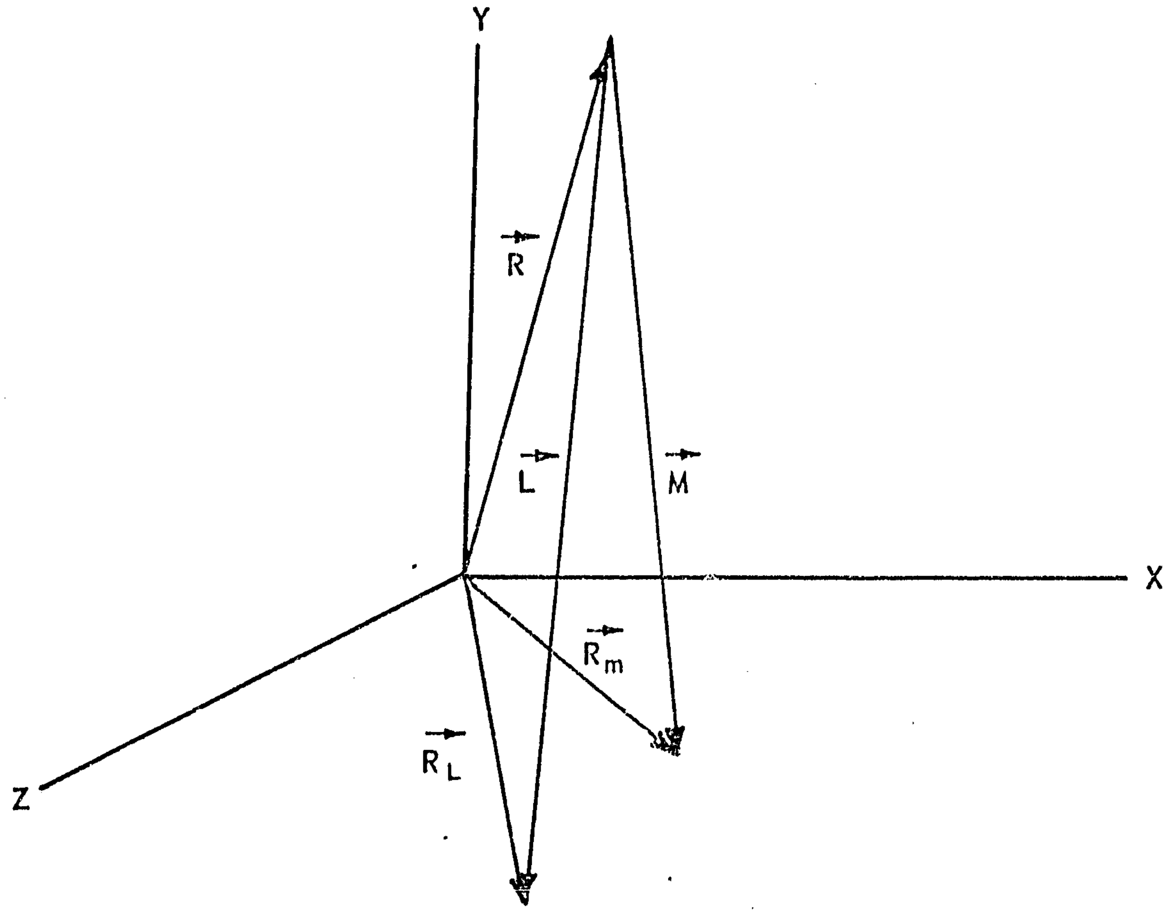


Figure 3

$$\cos(\vec{L},\vec{Z}) = \vec{L}_z/|\vec{L}|$$

$$\cos(\vec{M},\vec{X}) = \vec{M}_x/|\vec{M}|$$

$$\cos(\vec{M},\vec{Y}) = \vec{M}_y/|\vec{M}|$$

$$\cos(\vec{M},\vec{Z}) = \vec{M}_z/|\vec{M}|$$

Where $\vec{L} = L_x\vec{i} + L_y\vec{j} + L_z\vec{k}$, $\vec{M} = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$.

If $\vec{R}_L = X_L\vec{i} + Y_L\vec{j} + Z_L\vec{k}$, $\vec{R}_M = X_M\vec{i} + Y_M\vec{j} + Z_M\vec{k}$

$\vec{R} = X\vec{i} + Y\vec{j} + Z\vec{k}$, then by equation (8)

$$\cos(\vec{L},\vec{X}) = (X - X_L)/L \qquad \cos(\vec{M},\vec{X}) = (X - X_M)/M \tag{10}$$

$$\cos(\vec{L},\vec{Y}) = (Y - Y_L)/L \qquad \cos(\vec{M},\vec{Y}) = (Y - Y_M)/M$$

$$\cos(\vec{L},\vec{Z}) = (Z - Z_L)/L \qquad \cos(\vec{M},\vec{Z}) = (Z - Z_M)/M$$

where: $L = |\vec{L}| = \sqrt{(X - X_L)^2 + (Y - Y_L)^2 + (Z - Z_L)^2}$

$$M = |\vec{M}| = \sqrt{(X - X_L)^2 + (Y - Y_L)^2 + (Z - Z_L)^2} \ .$$

Having thus obtained the orientations of $\vec{L}$, $\vec{M}$ with respect to XYZ, we can now derive the orientations of $\vec{L}$, $\vec{M}$ with respect to $\vec{B}_L$, $\vec{B}_M$ respectively:

$$\cos(\vec{L},\vec{B}_L) = \vec{L}\cdot\vec{B}_L/|\vec{L}|\cdot|\vec{B}_L| \tag{11}$$

$$\cos(\vec{M},\vec{B}_M) = \vec{M}\cdot\vec{B}_M/|\vec{M}|\cdot|\vec{B}_M| \ .$$

But

$$\vec{L}\cdot\vec{B}_L = L_x B_L^{(x)} + L_y B_L^{(y)} + L_z B_L^{(z)}$$

$$M\cdot B_M = M_x B_M^{(x)} + M_y B_M^{(y)} + M_z B_M^{(z)}$$

and hence from equations (4) and (9)

$$\cos(\vec{L},\vec{B}_L) = \cos(\vec{L},\vec{X})\cdot\cos(\vec{B}_L,\vec{X}) + \cos(\vec{L},\vec{Y})\cdot\cos(\vec{B}_L,\vec{Y}) + \cos(\vec{L},\vec{Z})\cdot\cos(\vec{B}_L,\vec{Z}) \tag{12}$$

$$\cos(\vec{M},\vec{B}_M) = \cos(\vec{M},\vec{X})\cdot\cos(\vec{B}_M,\vec{X}) + \cos(\vec{M},\vec{Y})\cdot\cos(\vec{B}_M,\vec{Y}) + \cos(\vec{M},\vec{Z})\cdot\cos(\vec{B}_M,\vec{Z}) \quad .$$

The symbolic notation for these direction cosines is

$$\ell \equiv \cos(\vec{L},\vec{B}_L) \qquad\qquad m \equiv \cos(\vec{M},\vec{B}_M) \quad .$$

The AZUSA system normally measures $\ell, m, \dot{\ell}, \dot{m}$ and a slant range from some point (other than the centers of $\vec{B}_L, \vec{B}_M$) on the system, and an associated range rate. To derive this range, let $\vec{R}_5$ extend from the origin to range measuring point and let $\vec{R}_R$ extend from the vehicle to the range measuring point.

Then by the reasoning of equation (8),

$$\vec{R}_R = \vec{R} - \vec{R}_5 \tag{13}$$

where $\vec{R}$ is as in equation (8). Upon the assumption that

$$\vec{R}_5 = X_5\vec{i} + Y_5\vec{j} + Z_5\vec{k}$$

$$\vec{R}_R = (X - X_5)\vec{i} + (Y - Y_5)\vec{j} + (Z - Z_5)\vec{k} \quad . \tag{14}$$

To simplify the derivation of the expressions for the rates mentioned previously, let

$$a_L \equiv \cos(\vec{L},\vec{X}) = (X - X_L)/L \qquad a_M \equiv \cos(\vec{M},\vec{X}) = (X - X_M)/M$$

$$b_L \equiv \cos(\vec{L},\vec{Y}) = (Y - Y_L)/L \qquad b_M \equiv \cos(\vec{M},\vec{Y}) = (Y - Y_M)/M$$

$$c_L \equiv \cos(\vec{L},\vec{Z}) = (Z - Z_L)/L \qquad c_M \equiv \cos(\vec{M},\vec{Z}) = (Z - Z_M)/M$$

$$d_L \equiv \cos(\vec{B}_L,\vec{X}) \qquad d_M \equiv \cos(\vec{B}_M,\vec{X})$$

$$e_L \equiv \cos(\vec{B}_L,\vec{Y}) \qquad e_M \equiv \cos(\vec{B}_M,\vec{Y})$$

$$f_L \equiv \cos(\vec{B}_L,\vec{Z}) \qquad f_M \equiv \cos(\vec{B}_M,\vec{Z}) \quad .$$

Then, by equation (12)

$$\ell = a_L d_L + b_L e_L + c_L f_L \qquad m = a_M d_M + b_M e_M + c_M f_M \quad . \tag{15}$$

Now d_L, d_M, e_L, e_M, f_L, f_M are all constant and hence have zero derivatives with respect to any variable. Consequently,

$$\dot{\ell} = d_L \dot{a}_L + e_L \dot{b}_L + f_L \dot{c}_L \qquad \dot{m} = d_M \dot{a}_M + e_M \dot{b}_M + f_M \dot{c}_M \quad . \tag{16}$$

All that remains then is to evaluate $\dot{a}_L$, $\dot{b}_L$, $\dot{c}_L$, $\dot{a}_M$, $\dot{b}_M$, $\dot{c}_M$.

Now

$$\dot{a}_L = \left(\frac{\partial a_L}{\partial X}\right)\dot{X} + \left(\frac{\partial a_L}{\partial Y}\right)\dot{Y} + \left(\frac{\partial a_L}{\partial Z}\right)\dot{Z} \tag{17}$$

$$\dot{a}_M = \left(\frac{\partial a_M}{\partial X}\right)\dot{X} + \left(\frac{\partial a_M}{\partial Y}\right)\dot{Y} + \left(\frac{\partial a_M}{\partial Z}\right)\dot{Z}$$

$$\dot{b}_L = \left(\frac{\partial b_L}{\partial X}\right)\dot{X} + \left(\frac{\partial b_L}{\partial Y}\right)\dot{Y} + \left(\frac{\partial b_L}{\partial Z}\right)\dot{Z}$$

$$\dot{b}_M = \left(\frac{\partial b_M}{\partial X}\right)\dot{X} + \left(\frac{\partial b_M}{\partial Y}\right)\dot{Y} + \left(\frac{\partial b_M}{\partial Z}\right)\dot{Z}$$

$$\dot{c}_L = \left(\frac{\partial c_L}{\partial X}\right)\dot{X} + \left(\frac{\partial c_L}{\partial Y}\right)\dot{Y} + \left(\frac{\partial c_L}{\partial Z}\right)\dot{Z}$$

$$\dot{c}_M = \left(\frac{\partial c_M}{\partial X}\right)\dot{X} + \left(\frac{\partial c_M}{\partial Y}\right)\dot{Y} + \left(\frac{\partial c_M}{\partial Z}\right)\dot{Z}$$

and it is easily verified that

$$\frac{\partial a_L}{\partial X} = \frac{(1-a_L^2)}{L} \qquad \frac{\partial a_L}{\partial Y} = \frac{-a_L b_L}{L} \qquad \frac{\partial a_L}{\partial Z} = \frac{-a_L c_L}{L} \tag{18}$$

$$\frac{\partial b_L}{\partial X} = \frac{-a_L b_L}{L} \qquad \frac{\partial b_L}{\partial Y} = \frac{(1-b_L^2)}{L} \qquad \frac{\partial b_L}{\partial Z} = \frac{-b_L c_L}{L}$$

$$\frac{\partial c_L}{\partial X} = \frac{-a_L c_L}{L} \qquad \frac{\partial c_L}{\partial Y} = \frac{-b_L c_L}{L} \qquad \frac{\partial c_L}{\partial Z} = \frac{(1-c_L^2)}{L}$$

$$\left(\frac{\partial a_M}{\partial X}\right) = \frac{(1-a_M^2)}{M} \qquad \left(\frac{\partial a_M}{\partial Y}\right) = \frac{-a_M b_M}{M} \qquad \left(\frac{\partial a_M}{\partial X}\right) = \frac{-a_M c_M}{M}$$

$$\left(\frac{\partial b_M}{\partial X}\right) = \frac{-a_M b_M}{M} \qquad \left(\frac{\partial b_M}{\partial Y}\right) = \frac{(1-b_M^2)}{M} \qquad \left(\frac{\partial b_M^2}{\partial Z}\right) = \frac{-b_M c_M}{M}$$

$$\left(\frac{\partial c_M}{\partial X}\right) = \frac{-a_M c_M}{M} \qquad \left(\frac{\partial c_M}{\partial Y}\right) = \frac{-b_M c_M}{M} \qquad \left(\frac{\partial c_M}{\partial Z}\right) = \frac{(1-c_M^2)}{M}$$

Substitution of equations (17) and (18) into equation (16) yields the desired expressions for ℓ, m.

The range rate $\dot{R}_R$ is the rate of change of the magnitude of the range vector $\vec{R}_R$ in the direction of the velocity vector $\vec{V} = X\vec{i} + Y\vec{j} + Z\vec{k}$.

This is so since the vector representing the rate of change of $|\vec{R}_R|$ has a maximum component in the direction tangent to the flight path at (X,Y,Z), i.e., in the direction of $\vec{V}$. The space rate of change of $|\vec{R}_R|$ is given by

$$\nabla|\vec{R}_R| = \frac{\partial|\vec{R}_R|\,\vec{i}}{\partial X} + \frac{\partial|\vec{R}_R|\,\vec{j}}{\partial Y} + \frac{\partial|\vec{R}_R|\,\vec{k}}{\partial Z} \quad . \tag{19}$$

Then the range rate is given by

$$\dot{R}_R = \nabla|\vec{R}_R| \cdot \vec{V} \tag{20}$$

which upon expansion is

$$\dot{R}_R = \frac{\partial R_R}{\partial X} \dot{X} + \frac{\partial R_R}{\partial Y} \dot{Y} + \frac{\partial R_R}{\partial Z} \dot{Z} \tag{21}$$

where $R_R = \left|\vec{R}_R\right|$. Since

$$\frac{\partial R_R}{\partial X} = \frac{X-X_5}{R_R} \qquad \frac{\partial R_R}{\partial Y} = \frac{Y-Y_5}{R_R} \qquad \frac{\partial R_R}{\partial Z} = \frac{Z-Z_5}{R_R} \quad .$$

Then equation (21) becomes

$$\dot{R}_R = \frac{(X-X_5)\dot{X}}{R_R} + \frac{(Y-Y_5)\dot{Y}}{R_R} + \frac{(Z-Z_5)\dot{Z}}{R_R} \quad .$$

NOTE: The derivation of the equations used to transform geographic data to cartesian data can be found in Reference 1.

B. MATHEMATICS OF THE GDOP DETERMINATION

GDOP determination is the problem of determining to what degree errors in parameter measurements are carried through non-orthogonal transformations. The solution to the problem treated here is very specialized to simplify computations. In particular, it will be assumed that velocity contributes no error to position, the effect of which assumptions will be seen further on. The general development of a GDOP determination scheme will not be presented here. Also, see Reference 2.

Let S_1 be the position covariance matrix, which is a diagonal matrix consisting of the variances σ^2_{ℓ}, σ^2_{m}, σ^2_{R} (this presumes independence of errors for ℓ, m, R). Let $\overline{B}$ be the matrix of partial derivatives of ℓ, m, R with respect to X,Y,Z. Then the position covariance matrix $\overline{S}_2$ for X,Y,Z (i.e., the matrix indicating the extent of propagated error) is given by

$$\overline{S}_2 = \left[\overline{B}^T \overline{S}_1^{-1} \overline{B}\right]^{-1} \qquad (23)$$

Then the parameter variances for X,Y,Z are the square roots of the diagonal elements of S_2, i.e.,

$$\sigma_x = S^{(2)}_{11} \qquad \sigma_y = S^{(2)}_{22} \qquad \sigma_z = S^{(2)}_{33} \qquad (24)$$

where $\quad \overline{S}_2 = \left[S^{(2)}_{ij}\right]^{3}_{ij} = 1 \quad .$

The case for velocity is similar, except that the partial derivative matrix $\overline{B}$ contains the derivatives of $\dot{\ell}$, $\dot{m}$, $\dot{R}$ by $\dot{X}$, $\dot{Y}$, $\dot{Z}$, and in the place of $\overline{S}_1$, the covariance matrix, $\overline{S}_3$, for $\dot{\ell}$, $\dot{m}$, $\dot{R}$ is substituted, yielding the covariance matrix, S_4, for $\dot{X}$, $\dot{Y}$, $\dot{Z}$.

Now from equations (15) and (18) in paragraph A,

$$\frac{\partial \ell}{\partial X} = \frac{\partial a_L}{\partial X} d_L + \frac{\partial b_L}{\partial X} e_L + \frac{\partial c_L}{\partial X} f_L \equiv A_L \tag{25}$$

$$= \frac{(1-a_L^2)}{L} d_L - \frac{a_L b_L e_L}{L} - \frac{a_L c_L f_L}{L}$$

$$\frac{\partial \ell}{\partial Y} = \frac{-a_L b_L d_L}{L} + \frac{(1-b_L^2)e_L}{L} - \frac{b_L c_L f_L}{L} \equiv B_L$$

$$\frac{\partial \ell}{\partial Z} = \frac{-a_L c_L d_L}{L} - \frac{b_L c_L f_L}{L} + \frac{(1-c_L^2)f_L}{L} \equiv C_L$$

$$\frac{\partial m}{\partial X} = \frac{(1-a_M^2)d_M}{M} - \frac{a_M b_M e_M}{M} - \frac{a_M c_M f_M}{M} \equiv A_M$$

$$\frac{\partial m}{\partial Y} = \frac{-a_M b_M d_M}{M} + \frac{(1-b_M^2)e_M}{M} - \frac{b_M c_M f_M}{M} \equiv B_M$$

$$\frac{\partial M}{\partial Z} = \frac{-a_M c_M d_M}{M} - \frac{b_M c_M e_M}{M} + \frac{(1-c_M^2)f_M}{M} \equiv C_M$$

and also from the statement preceding equation (22) in paragraph A

$$\frac{\partial R_R}{\partial X} = \frac{X-X_5}{R_R} \cdot A_R \qquad \frac{\partial R_R}{\partial Y} = \frac{Y-Y_5}{R_R} \equiv B_R \qquad \frac{\partial R_R}{\partial Z} = \frac{Z-Z_5}{R_R} \equiv C_R \tag{26}$$

The computed values of equations (25) and (26) are substituted into equation (23) giving

$$\bar{S}_2 = \left\{ \begin{bmatrix} A_L & A_M & A_R \\ B_L & B_M & B_R \\ C_L & C_M & C_R \end{bmatrix} \cdot \begin{bmatrix} \sigma_\ell^{-2} & 0 & 0 \\ 0 & \sigma_m^{-2} & 0 \\ 0 & 0 & \sigma_R^{-2} \end{bmatrix} \cdot \begin{bmatrix} A_L & B_L & C_L \\ A_M & B_M & C_M \\ A_R & B_R & C_R \end{bmatrix} \right\}^{-1} \tag{27}$$

When the matrix product in the brackets is expanded, a matrix A is obtained whose elements are

$$A_{11} = \frac{A_L^2}{\sigma_\ell^2} + \frac{A_M^2}{\sigma_m^2} + \frac{A_R^2}{\sigma_R^2} \tag{28}$$

$$A_{22} = \frac{B_L^2}{\sigma_\ell^2} + \frac{B_M^2}{\sigma_m^2} + \frac{B_R^2}{\sigma_R^2}$$

$$A_{33} = \frac{C_L^2}{\sigma_\ell^2} + \frac{C_M^2}{\sigma_m^2} + \frac{C_R^2}{\sigma_R^2}$$

$$A_{12} = \frac{A_L B_L}{\sigma_\ell^2} + \frac{A_M B_M}{\sigma_m^2} + \frac{A_R B_R}{\sigma_R^2}$$

$$A_{13} = \frac{A_L C_L}{\sigma_\ell^2} + \frac{A_M C_M}{\sigma_m^2} + \frac{A_R C_R}{\sigma_R^2}$$

$$A_{21} = A_{12}$$

$$A_{23} = \frac{B_L C_L}{\sigma_\ell^2} + \frac{B_M C_M}{\sigma_m^2} + \frac{B_R C_R}{\sigma_R^2}$$

$$A_{31} = A_{13}$$

$$A_{32} = A_{23} \quad .$$

The inverse of $\overline{A}$, to yield $\overline{S}_2$, is then obtained by using the cofactor inversion method.

In general it is not true that

$$\frac{\partial p}{\partial X} = \frac{\partial \dot{p}}{\partial \dot{X}}\ ,$$

for any parameter p, but in the present situation it is true that

$$\frac{\partial \ell}{\partial X} = \frac{\partial \dot{\ell}}{\partial \dot{X}} \qquad \frac{\partial m}{\partial X} = \frac{\partial \dot{m}}{\partial \dot{X}} \qquad \frac{\partial R_R}{\partial \dot{X}} = \frac{\partial \dot{R}_R}{\partial \dot{X}}\ , \text{ etc.}$$

To see that this is so, recall that (from equation (16) paragraph A)

$$\ell = d_L \dot{a}_L + e_L \dot{b}_L + f_L \dot{c}_L \qquad m = d_M \dot{a}_M + e_M \dot{b}_M + f_M \dot{c}_M$$

and that from equations (17) and (18) this yields

$$\ell = \left[\frac{(1-a_L^2)d_L}{L} - \frac{a_L b_L e_L}{L} - \frac{a_L c_L f_L}{L}\right]\dot{X} \tag{29}$$

$$+ \left[\frac{-a_L b_L d_L}{L} + \frac{(1-b_L^2)e_L}{L} - \frac{b_L c_L f_L}{L}\right]\dot{Y}$$

$$+ \left[\frac{-a_L c_L d_L}{L} - \frac{b_L c_L e_L}{L} + \frac{(1-c_L^2)f_L}{L}\right]\dot{Z}$$

$$m = \left[\frac{(1-a_M^2)d_M}{M} - \frac{a_M b_M e_M}{M} - \frac{a_M c_M f_M}{M}\right]\dot{X}$$

$$+ \left[\frac{-a_M b_M d_M}{M} + \frac{(1-b_M^2)e_M}{M} - \frac{b_M c_M f_M}{M}\right]\dot{Y}$$

$$+ \left[\frac{-a_M c_M d_M}{M} - \frac{b_M c_M e_M}{M} + \frac{(1-c_M^2)f_M}{M}\right]\dot{Z} \quad .$$

Upon differentiating equation (29) by $\dot{X}$, $\dot{Y}$, $\dot{Z}$, it is seen that

$$\frac{\partial \dot{\ell}}{\partial \dot{X}} = \frac{(1-a_L^2)d_L}{L} - \frac{a_L b_L e_L}{L} - \frac{a_L c_L f_L}{L} \tag{30}$$

$$\frac{\partial \dot{\ell}}{\partial \dot{Y}} = \frac{-a_L b_L d_L}{L} + \frac{(1-b_L^2)}{L} - \frac{b_L c_L f_L}{L}$$

$$\frac{\partial \dot{\ell}}{\partial \dot{Z}} = \frac{-a_L c_L d_L}{L} - \frac{b_L c_L e_L}{L} + \frac{(1-b_L^2)}{L}$$

$$\frac{\partial \dot{m}}{\partial \dot{X}} = \frac{(1-a_M^2)d_M}{M} - \frac{a_M b_M e_M}{M} - \frac{a_M c_M f_M}{M}$$

$$\frac{\partial \dot{m}}{\partial \dot{Y}} = \frac{-a_M b_M d_M}{M} + \frac{(1-b_M^2)e_M}{M} - \frac{b_M c_M f_M}{M}$$

$$\frac{\partial \dot{m}}{\partial \dot{Z}} = \frac{-a_M c_M d_M}{M} - \frac{b_M c_M e_M}{M} + \frac{(1-c_M^2)f_M}{M} \quad .$$

Upon examination of equations (30) and (24), it is seen that

$$\frac{\partial \dot{\ell}}{\partial \dot{X}} = A_M \qquad \frac{\partial \dot{\ell}}{\partial \dot{Y}} = B_L \qquad \frac{\partial \dot{\ell}}{\partial \dot{Z}} = C_L \tag{31}$$

$$\frac{\partial \dot{m}}{\partial \dot{X}} = A_M \qquad \frac{\partial \dot{m}}{\partial \dot{Y}} = B_M \qquad \frac{\partial \dot{m}}{\partial \dot{Z}} = C_M \quad .$$

Recalling equations (22) and (26),

$$\frac{\partial \dot{R}_R}{\partial \dot{X}} = \frac{(X-X_5)}{R_R} = A_R \qquad \frac{\partial \dot{R}_R}{\partial \dot{Y}} = \frac{(Y-Y_5)}{R_R} = B_R \tag{32}$$

$$\frac{\partial \dot{R}_R}{\partial \dot{Z}} = \frac{(Z-Z_R)}{R_R} = C_R \quad .$$

Consequently, for computation purposes, once $\overline{B}$ has been computed for position, it is saved and used in the velocity solution, since $\overline{B}$ is the same in both cases.

A computation is also made which propagates $\dot{X}, \dot{Y}, \dot{Z}$ errors into the magnitude $|\vec{V}|$ of the velocity vector $\vec{V} = \dot{X}\vec{i} + \dot{Y}\vec{j} + \dot{Z}\vec{k}$. Note that this is a just-determined condition since $\dot{X}, \dot{Y}, \dot{Z}$ completely determine $\vec{V}$. In such a case the covariance matrix $\bar{S}_5$ for V is given by

$$\bar{S}_5 = \bar{C}\ \bar{S}_4\ \bar{C}^T \tag{33}$$

where $\bar{S}_4$ is the covariance matrix of $\dot{X}, \dot{Y}, \dot{Z}$, as in the remarks preceding equation (24), and $\bar{C}$ is the matrix of partial derivatives of $V = |\vec{V}|$ with respect to $\dot{X}, \dot{Y}, \dot{Z}$.

It will be seen that

$$\frac{\partial V}{\partial \dot{X}} = \frac{\dot{X}}{V} \qquad \frac{\partial V}{\partial \dot{Y}} = \frac{\dot{Y}}{V} \qquad \frac{\partial V}{\partial \dot{Z}} = \frac{\dot{Z}}{V} \tag{34}$$

and thus $\bar{C} = \begin{pmatrix} \frac{\dot{X}}{V} & \frac{\dot{Y}}{V} & \frac{\dot{Z}}{V} \end{pmatrix}$. Then equation (33) becomes

$$\bar{S}_5 = \begin{pmatrix} \frac{\dot{X}}{V} & \frac{\dot{Y}}{V} & \frac{\dot{Z}}{V} \end{pmatrix} \begin{bmatrix} \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\dot{y}} & \sigma_{\dot{x}\dot{z}} \\ \sigma_{\dot{x}\dot{y}} & \sigma_{\dot{y}}^2 & \sigma_{\dot{y}\dot{z}} \\ \sigma_{\dot{x}\dot{z}} & \sigma_{\dot{y}\dot{z}} & \sigma_{\dot{z}}^2 \end{bmatrix} \begin{bmatrix} \frac{\dot{X}}{V} \\ \frac{\dot{Y}}{V} \\ \frac{\dot{Z}}{V} \end{bmatrix} \tag{35}$$

which on expansion becomes

$$\bar{S}_5 = \left(\frac{1}{V^2}\right) \left[\dot{X}^2\sigma_{\dot{x}}^2 + \dot{Y}^2\sigma_{\dot{y}}^2 + \dot{Z}\sigma_{\dot{z}}^2 + 2\dot{X}\dot{Y}\sigma_{\dot{x}\dot{y}} + 2\dot{X}\dot{Z}\sigma_{\dot{x}\dot{z}} + 2\dot{Y}\dot{Z}\sigma_{\dot{y}\dot{z}}\right] \tag{36}$$

Since $\overline{S}_5$ has only one element, (37)

$$\sigma_V = \frac{\sqrt{\dot{X}^2\sigma_{\dot{x}}^2 + \dot{Y}^2\sigma_{\dot{y}}^2 + \dot{Z}^2\sigma_{\dot{z}}^2 + 2\dot{X}\dot{Y}\sigma_{\dot{x}\dot{y}} + 2\dot{X}\dot{Z}\sigma_{\dot{x}\dot{z}} + 2\dot{Y}\dot{Z}\sigma_{\dot{a}\dot{z}}}}{V}$$

SECTION V
COMPUTER PROGRAM OPERATING INSTRUCTIONS

A. PROGRAM DESCRIPTION

1.	Program Identification	AZGD2
2.	Computer	GE-635
3.	Program Library Number	1016
4.	Type of Coding	Fortran IV
5.	Core Storage	6666

B. USAGE AND LIMITATIONS

This program computes GDOPS (accuracies) for a standard KSC theoretical trajectory based on an AZUSA (or AZUSA-like) system; that is, a tracking system with two intersecting baselines.

C. PROGRAM INPUT FORMAT

1. Tape Input

Standard KSC trajectory tape, or with the following format will be used:

Word No.	Entry	
1	Time	(sec)
2-4	X, Y, Z	(mtrs)
5-7	$\dot{X}$, $\dot{Y}$, $\dot{Z}$	(mtrs/sec)

2. Card Input

a. Deck Setup

$	SNUMB	Operations Supplied (OS)
$	IDENT	Requester Supplied (RS)
$	OPTION FORTRAN	RS
	PROGRAM DECK	OS
$	EXECUTE	RS
$	LIMITS	RS
$	TAPE 03	RS
$	FFILE 03	RS
$	TAPE 04	RS
$	INCODE IBMF	RS
	DATA CARDS	RS
$	ENDJOB	RS

***EOF

b. Card Format

Card	Columns	Information	Format	Remarks
1	1		$.	Control card
	8-12		IDENT	
	16-20	Program ID	AZGD2	
	21		,	Comma
	22-27	Requester's control number	XXXXXX	
	28		,	Comma
	29-31	Requester's initials	XX	
	32		,	Comma
	33-34	Project code	XX	
	35			
	36-37	Rerun code	XX	
	38			
	39-48	Work order number	XXXXXXXXXX	
2	1		$	Control card
	8-14		EXECUTE	
3	1		$	Control card
	8-13		LIMITS	
	16-	Maximum run time, hr/100		
			,	Comma

Card	Columns	Information	Format	Remarks
3 (Cont)		Core storage	8000	
			,,	Commas
		Maximum number of lines to print		≤10000
4	1		$	Control card
	8-11		TAPE	
	16-17	File code	03	Zero-three
	18		,	Comma
	19-21	Logical unit designator	AIR	If input tape is to be returned to requester, replace R with D.
	22-23		,,	Commas
	24-28	File serial number		99999 if unknown
	29-30		,,	Commas
	31-42	File name for external use		Information on tape label
5	1		$	Control card (only present for tape input)
	8-12		FFILE	
	16-17	File code	03	Zero-three
	18		,	Comma

Card	Columns	Information	Format	Remarks
5 (Cont)	19-24		NSTDBL	No standard labels
	25		,	Comma
	26-34		BUFSIZ/54	54 words per record
	35		,	Comma
	36-44		FIXLNG/54	54 words per record
	45		,	Comma
	46-51		NØSRLS	No block serials
6	1		$	Control card
	8-11		TAPE	
	16-17	File code	04	Zero-four
	18		,	Comma
	19-21	Logical unit designator	B1R	If output tape is to be saved, use S or D for R.

If output tape is needed the following fields must be used on Card 5:

5	22-25		,,,,	Commas
	26-37	File for external use		
7	1		$	Control card
	8-13		INCØDE	
	16-19		IBMF	
8	1-5	Program ID	AZGD2	Comment card
	6-80	Anything		

Card	Columns	Information	Format	Remarks
9	1	Coordinate system of output	X	0=output in coordinate system (X=downrange Y=up Z=crossrange) 1=output in Apollo Saturn Standard Coordinate System (Z=downrange X=up Y=crossrange)
10	1-15	a of spheroid	±0.XXXXXXXXE±XX	Meters
	16-30	b of spheroid	±0.XXXXXXXXE±XX	Meters
	31-45	Azimuth of X-axis	±0.XXXXXXXXE±XX	Degrees
11-16	1-9			
	10-12	φj, Degree	±XX	Trajectory origin
		j=0: trajectory origin		Latitude
		j=1:L1 Coordinate		L Baseline
		j=2:L2 Coordinate		Latitude
		j=3:M1 Coordinate		M Baseline
		j=4:M2 Coordinate		Latitudes
		j=5: Origin the R parameter		R origin latitude

Card	Columns	Information	Format	Remarks
11-16 (Cont)	14-15	φj, Minute	XX	
	17-24	φj, Second	XX.XXXXX	
	27-30	λj, Degree	±XXX	Longitude
	32-33	λj, Minute	XX	Longitude
	35-42	λj, Second	XX.XXXXX	
	45-56	hj, Meters	±XXXX.XXXXXX	Height
	57			
	58	Spheroid	X	F, C, W or K
	59			
	60-80	ID of location j		

Example:
(Card 11) Pad Coordinate
(Card 12) L1 Coordinate
(Card 13) L2 Coordinate
(Card 14) M1 Coordinate
(Card 15) M2 Coordinate
(Card 16) R Coordinate

Cards 11-16 are all coordinate cards and must be in the order shown above.

Card	Columns	Information	Format	Remarks
17	1-15	σ_{ℓ}	±0.XXXXXXXXE±XX	L-direction cosine accuracy
	16-30	σ_m	±0.XXXXXXXXE±XX	M-direction cosine accuracy
	31-45	σ_R	±0.XXXXXXXXE±XX	Range accuracy (mtrs)
	46-60	$\sigma_{\dot{\ell}}$	±0.XXXXXXXXE±XX	L-rate accuracy
	61-75	$\sigma_{\dot{m}}$	±0.XXXXXXXXE±XX	M-rate accuracy

Card	Columns	Information	Format	Remarks
18	1-15	$\sigma_{\dot{R}}$	±0.XXXXXXXXE±XX	Range-rate accuracy (mtrs/sec)
19	1-15	Start time	±0.XXXXXXXXE±XX	Second
	16-30	Stop time	±0.XXXXXXXXE±XX	Second
20	1		$	Control card
	8-13		ENDJOB	
21	1-6		***EOF	End of file

NOTE: The program normally calls for a Fortran binary output tape on logical File 04. If the tape is not needed, leave Columns 22-80 blank on Card 6.

D. PROGRAM OUTPUT FORMAT

The output data are a printer output listing.

E. SAMPLE TEST CASE

1. Input

a. Test One

```
$       IDENT     AZGD2,CP3610,SP ,05 TO 7PB0011016
$       OPTION    FORTRAN
        PROGRAM DECK
$       EXECUTE
$       LIMITS    5,10000,0,10000
$       TAPE      03,A1R,,1494 ,,KSC FORMAT
$       FFILE     03,NSTDLB,BUFSIZ/54,FIXLNG/54,NOSRLS
$       TAPE      04,B1R,,,SCRATCH
$       INCODE    IBMF
AZGD2 TEST OF MOD BY D  SHANKEN
1
+0.63781660E+07+0.63567843E+07+0.95200000E+02  +.866        PA 37B
          028 24 46.89898  -080 35 31.22706   10.207485
          028 24 49.71176  -080 35 33.06384   10.21804810
          028 24 49.11759  -080 35 30.55415   10.205599
          028 24 47.49314  -080 35 33.73718   10.207458
          028 24 48.30449  -080 35 32.14502   10.22
+0.20000000E-04+0.20000000E-04+0.50000000E+01+0.50000000E-06+0.50000000E-06
+0.10000000E+00
+0.00000000E+00+0.50000000E+02
$       ENDJOB
***EOF
```

AZGD2 TEST J. BY D. SCHANKEN

A M 0.63781660E 07 B M 0.63567843E 07 DR-AZ DEG 0.99200000E 02

LATITUDE	LONGITUDE	ALTITUDE	ID
28.31. 53.12640	-80.33. 54.60810	4.86600000	PA 378

LATITUDE	LONGITUDE	ALTITUDE	ID
28.24. 46.89898	-8. 3.031.2269900010.21093750		

LATITUDE	LONGITUDE	ALTITUDE	ID
28.24. 49.71176	-80.35. 33.06384	10.21804810	

LATITUDE	LONGITUDE	ALTITUDE	ID
28.24. 49.11759	-80.35. 30.55415	10.20559895	

LATITUDE	LONGITUDE	ALTITUDE	ID
28.24. 47.49314	-80.35. 33.73718	10.20745802	

LATITUDE	LONGITUDE	ALTITUDE	ID
28.24. 48.30449	-80.35. 32.14502	10.22000003	

SIG L 0.20000000E-04 SIG M 0.20000000E-04 SIG R M 0.50000000E 01
SIGDL 0.50000000E-06 SIGDM 0.50000000E-06 SIGDR M/S 0.10000000E 00

APOLLO SATURN STANDARD COORDINATE SYSTEM

Z=DOWNRANGE (DR); X=UP; Y=CROSSRANGE (90>DR)

b. Test Two

```
$ -     IDENT   AZGD2,CP3610,SP ,05 TO 7PB0011016
$       OPTION  FORTRAN
        PROGRAM DECK
$       EXECUTE
$       LIMITS  5,10000,0,10000
$       TAPE    03,A1R,,1494 ,,KSC FORMAT
$       FFILE   03,NSTDLB,BUFSIZ/54,FIXLNG/54,NOSRLS
$       TAPE    04,B1R,,,SCRATCH
$       INCODE  IBMF
AZGD2 TEST OF MOD BY D  SHANKEN
0
+0.63781660E+07+0.63567843E+07+0.95200000E+02 +.866          PA 37B
               028 24 46.89898  -080 35 31.22706   10.207485
               028 24 49.71176  -080 35 33.06384   10.21804810
               028 24 49.11759  -080 35 30.55415   10.205599
               028 24 47.49314  -080 35 33.73718   10.207458
               028 24 48.30449  -080 35 32.14502   10.22
+0.20000000E-04+0.20000000E-04+0.50000000E+01+0.50000000E-06+0.50000000E-06
+0.10000000E+00
+0.00000000E+00+0.50000000E+02
$       ENDJOB
***EOF
```

AZGD2 TE. OF MOD BY D SHANKEN

A M 0,63781660E 07 B M 0.63567843E 07 DR-AZ DEG 0,95200000E 02

LATITUDE	LONGITUDE	ALTITUDE	ID
28,31. 53.12640	-80,33. 54.60810	4.05600000	PA 378

LATITUDE	LONGITUDE	ALTITUDE	ID
28,24. 46,89898	-8, 3,031.2269900010.21093750		

LATITUDE	LONGITUDE	ALTITUDE	ID
28,24. 49,71176	-80,35. 33,06384	10.21804810	

LATITUDE	LONGITUDE	ALTITUDE	ID
28,24. 49,11759	-86,35. 30,55415	10.20559895	

LATITUDE	LONGITUDE	ALTITUDE	ID
28,24. 47,49314	-80,35. 33,73718	10,20745802	

LATITUDE	LONGITUDE	ALTITUDE	ID
28,24. 48,30449	-80,35. 32.14502	10,22000003	

SIG L 0,20000000E-04 SIG M 0,20000000E-04 SIG R M 0.30000000E 01
SIGDL 0,50000000E-06 SIGDM 0,50000000E-06 SIGDR M/S 0,10000000E 00

COORDINATE SYSTEM

X=DOWNRANGE (DR); Y=UP;Z=CROSSRANGE (90>DR)

2. Output

a. Tape Output Format

Word No.	Entry
1	Time (sec)
2-8	σ_x, σ_y, σ_z (mtrs)
	$\sigma_{\dot{x}}$, $\sigma_{\dot{y}}$, $\sigma_{\dot{z}}$, $\sigma_{\dot{V}}$ (mtrs/sec)

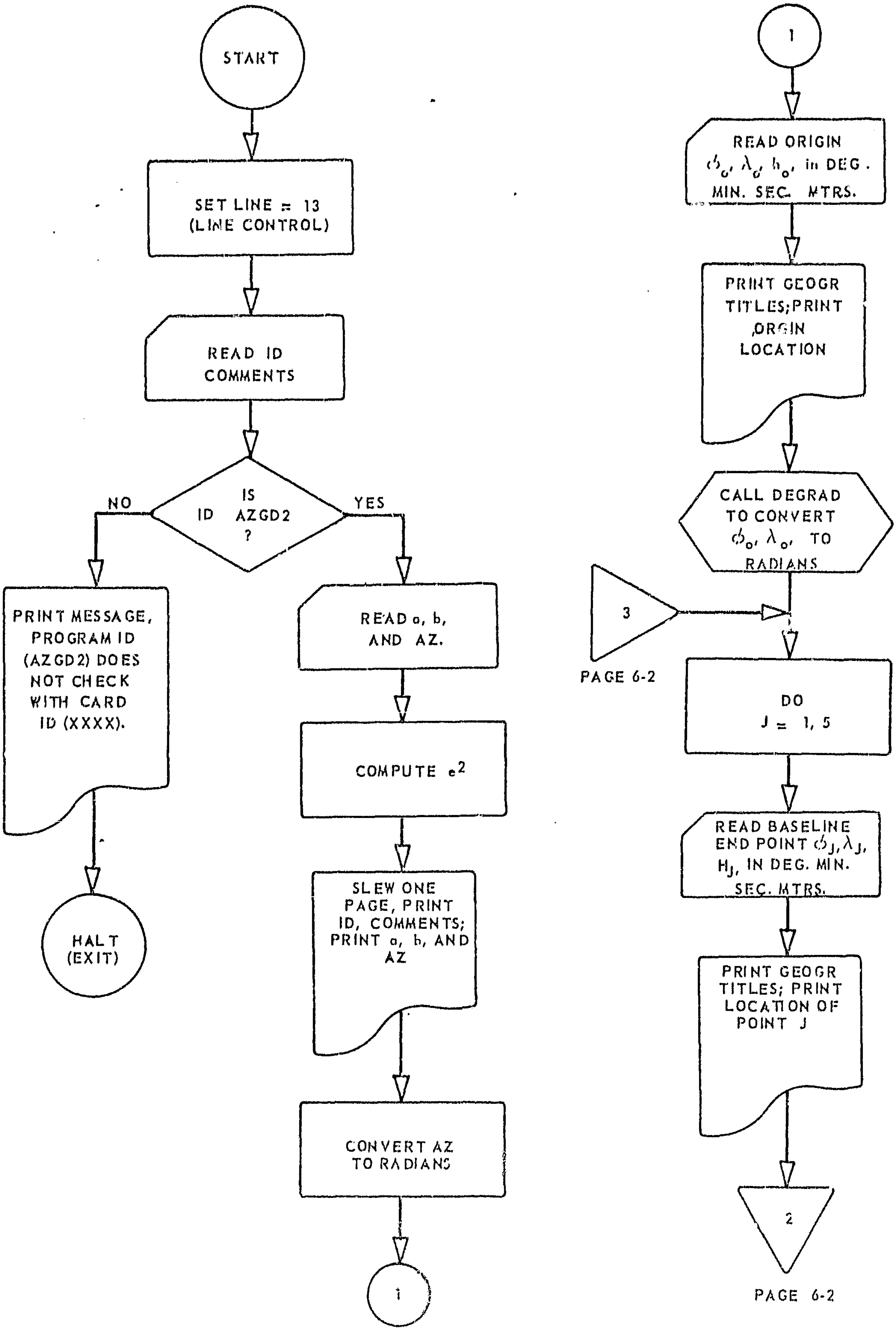
b. Test One

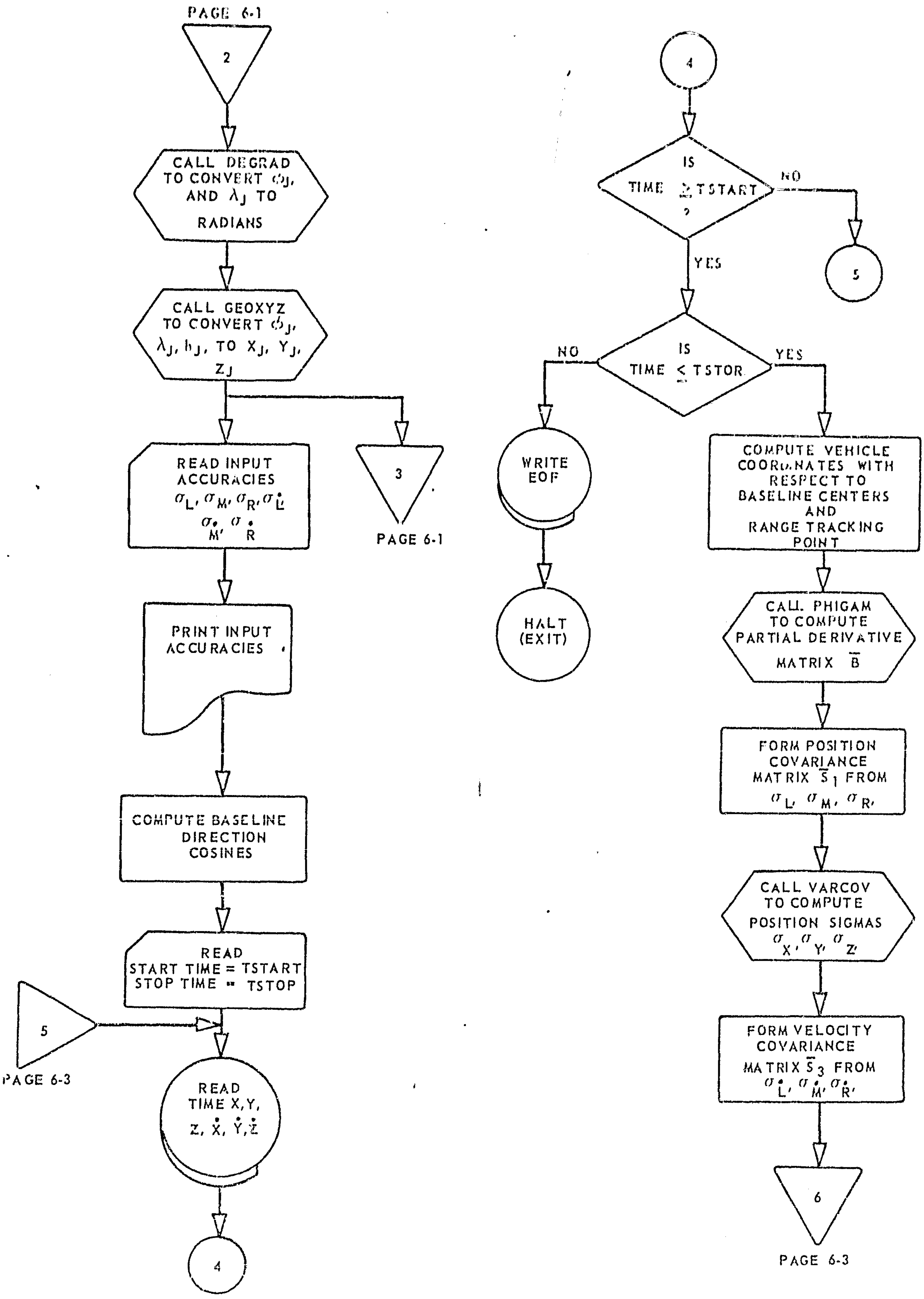
```
TIME SEC  0.                  SIG VV M/S   0.22071069E 04
 SIG Z M  0.96984651E 02      SIG DZ M/S   0.24249350E 01
 SIG X M  0.88302438E 05      SIG DX M/S,  0.22078628E 04
 SIG Y M  0.28355843E 03      SIGYDZ M/S   0.70695281E 01

TIME SEC  0.10000000E 01      SIG VV M/S   0.22108007E 04
 SIG Z M  0.10195760E 03      SIG DZ M/S   0.25509016E 01
 SIG X M  0.88364322E 05      SIG DX M/S,  0.22108105E 04
 SIG Y M  0.29071130E 03      SIGYDZ M/S   0.72730152E 01

TIME SEC  0.20000000E 01      SIG VV M/S   0.22114857E 04
 SIG Z M  0.11739844E 03      SIG DZ M/S   0.29352213E 01
 SIG X M  0.88451439E 05      SIG DX M/S,  0.22114899E 04
 SIG Y M  0.31274728E 03      SIGYDZ M/S   0.78190370E 01

TIME SEC  0.30000000E 01      SIG VV M/S   0.22167631E 04
 SIG Z M  0.14429137E 03      SIG DZ M/S   0.36090278E 01
 SIG X M  0.88627494E 05      SIG DX M/S,  0.22167632E 04
 SIG Y M  0.35116767E 03      SIGYDZ M/S   0.87831268E 01

TIME SEC  0.40000000E 01      SIG VV M/S   0.22257035E 04
 SIG Z M  0.18355824E 03      SIG DZ M/S   0.45879864E 01
 SIG X M  0.89046787E 05      SIG DX M/S,  0.22257023E 04
 SIG Y M  0.40753543E 03      SIGYDZ M/S   0.10185964E 02

TIME SEC  0.50000000E 01      SIG VV M/S   0.22482259E 04
 SIG Z M  0.23452763E 03      SIG DZ M/S   0.59123851E 01
 SIG X M  0.89180629E 05      SIG DX M/S,  0.22482240E 04
 SIG Y M  0.48002524E 03      SIGYDZ M/S   0.12101078E 02

TIME SEC  0.60000000E 01      SIG VV M/S   0.22476587E 04
 SIG Z M  0.30448053E 03      SIG DZ M/S   0.74994819E 01
 SIG X M  0.91255278E 05      SIG DX M/S,  0.22476564E 04
 SIG Y M  0.58264159E 03      SIGYDZ M/S   0.14350528E 02

TIME SEC  0.70000000E 01      SIG VV M/S   0.22580219E 04
 SIG Z M  0.38035312E 03      SIG DZ M/S   0.94512744E 01
 SIG X M  0.90870768E 05      SIG DX M/S,  0.22580196E 04
 SIG Y M  0.68958733E 03      SIGYDZ M/S   0.17135178E 02

TIME SEC  0.80000000E 01      SIG VV M/S   0.23748598E 04
 SIG Z M  0.48620792E 03      SIG DZ M/S   0.12304393E 02
 SIG X M  0.93842472E 05      SIG DX M/S,  0.23748580E 04
 SIG Y M  0.84459590E 03      SIGYDZ M/S   0.21373917E 02

TIME SEC  0.90000000E 01      SIG VV M/S   0.24317640E 04
 SIG Z M  0.60768546E 03      SIG DZ M/S   0.15385470E 02
 SIG X M  0.96048232E 05      SIG DX M/S,  0.24317635E 04
 SIG Y M  0.10204910E 04      SIGYDZ M/S   0.25836018E 02

TIME SEC  0.10000000E 02      SIG VV M/S   0.25525217E 04
 SIG Z M  0.78761069E 03      SIG DZ M/S   0.19460028E 02
 SIG X M  0.10326649E 06      SIG DX M/S,  0.25525237E 04
 SIG Y M  0.12875503E 04      SIGYDZ M/S   0.31825359E 02
```

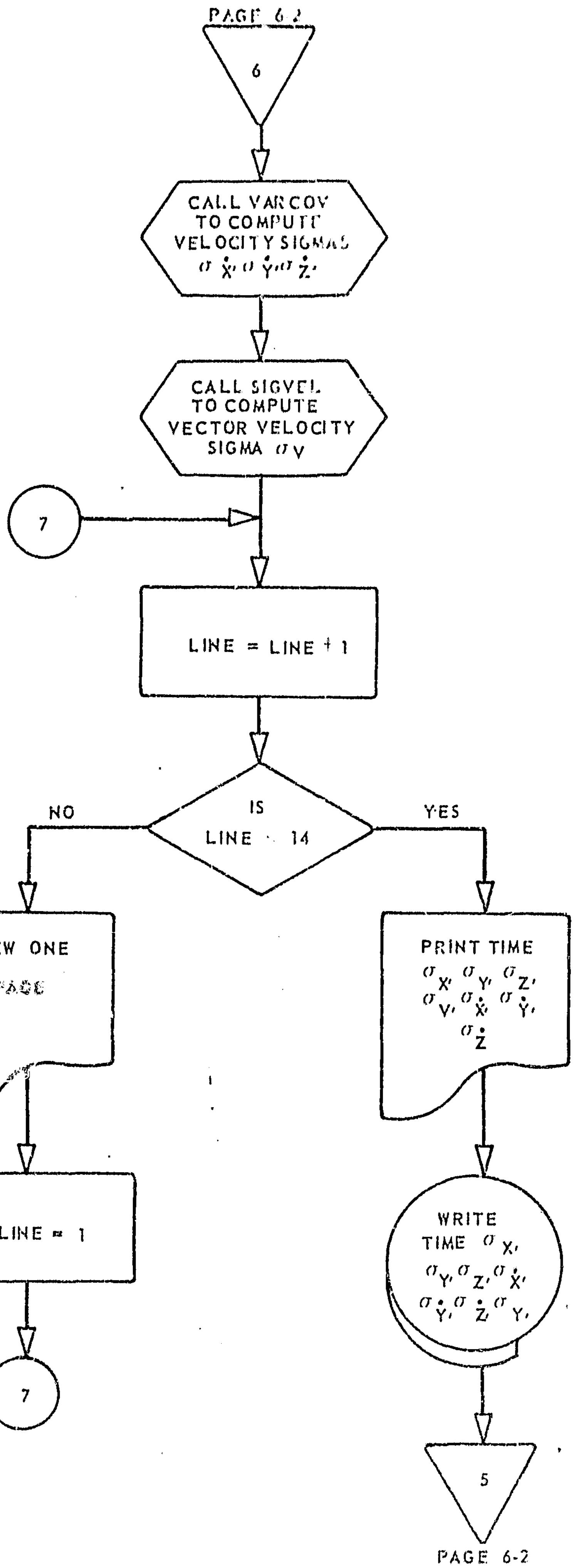
c. Test Two

```
TIME SEC   0,                      SIG VV M/S    0,22071069E 04
 SIG X M   0,96984651E 02          SIG DX M/S    0,24249350E 01
 SIG Y M   0,88302438E 05          SIG DY M/S,   0,22078628E 04
 SIG Z M   0,28355843E 03          SIG DZ M/S    0,70895281E 01

TIME SEC   0,10000000E 01          SIG VV M/S    0,22108007E 04
 SIG X M   0,10195760E 03          SIG DX M/S    0,25509016E 01
 SIG Y M   0,88364322E 05          SIG DY M/S,   0,22108185E 04
 SIG Z M   0,29071130E 03          SIG DZ M/S    0,72730152E 01

TIME SEC   0,20000000E 01          SIG VV M/S    0,22114857E 04
 SIG X M   0,11739844E 03          SIG DX M/S    0,29352213E 01
 SIG Y M   0,88451439E 05          SIG DY M/S,   0,22114899E 04
 SIG Z M   0,31274728E 03          SIG DZ M/S    0,78190370E 01

TIME SEC   0,30000000E 01          SIG VV M/S    0,22167631E 04
 SIG X M   0,14429137E 03          SIG DX M/S    0,36090278E 01
 SIG Y M   0,88627494E 05          SIG DY M/S,   0,22167632E 04
 SIG Z M   0,35116767E 03          SIG DZ M/S    0,87831268E 01

TIME SEC   0,40000000E 01          SIG VV M/S    0,22257035E 04
 SIG X M   0,18355824E 03          SIG DX M/S    0,45879864E 01
 SIG Y M   0,89046787E 05          SIG DY M/S,   0,22257023E 04
 SIG Z M   0,40753543E 03          SIG DZ M/S    0,10185964E 02

TIME SEC   0,50000000E 01          SIG VV M/S    0,22482259E 04
 SIG X M   0,23452763E 03          SIG DX M/S    0,59123851E 01
 SIG Y M   0,89180629E 05          SIG DY M/S,   0,22482240E 04
 SIG Z M   0,48002524E 03          SIG DZ M/S    0,12101078E 02

TIME SEC   0,60000000E 01          SIG VV M/S    0,22476587E 04
 SIG X M   0,30448053E 03          SIG DX M/S    0,74994819E 01
 SIG Y M   0,91255278E 05          SIG DY M/S,   0,22476564E 04
 SIG Z M   0,58264159E 03          SIG DZ M/S    0,14350528E 02

TIME SEC   0,70000000E 01          SIG VV M/S    0,22580219E 04
 SIG X M   0,38035312E 03          SIG DX M/S    0,94512744E 01
 SIG Y M   0,90870768E 05          SIG DY M/S,   0,22580196E 04
 SIG Z M   0,68958733E 03          SIG DZ M/S    0,17135178E 02

TIME SEC   0,80000000E 01          SIG VV M/S    0,23748598E 04
 SIG X M   0,48620792E 03          SIG DX M/S    0,12304393E 02
 SIG Y M   0,93842472E 05          SIG DY M/S,   0,23748580E 04
 SIG Z M   0,84459590E 03          SIG DZ M/S    0,21373917E 02

TIME SEC   0,90000000E 01          SIG VV M/S    0,24317640E 04
 SIG X M   0,60768546E 03          SIG DX M/S    0,15385470E 02
 SIG Y M   0,96048232E 05          SIG DY M/S,   0,24317635E 04
 SIG Z M   0,10204910E 04          SIG DZ M/S    0,25036818E 02

TIME SEC   0,10000000E 02          SIG VV M/S    0,25529217E 04
 SIG X M   0,78761069E 03          SIG DX M/S    0,19468028E 02
 SIG Y M   0,10326649E 06          SIG DY M/S,   0,25529237E 04
 SIG Z M   0,12875503E 04          SIG DZ M/S    0,31825359E 02
```

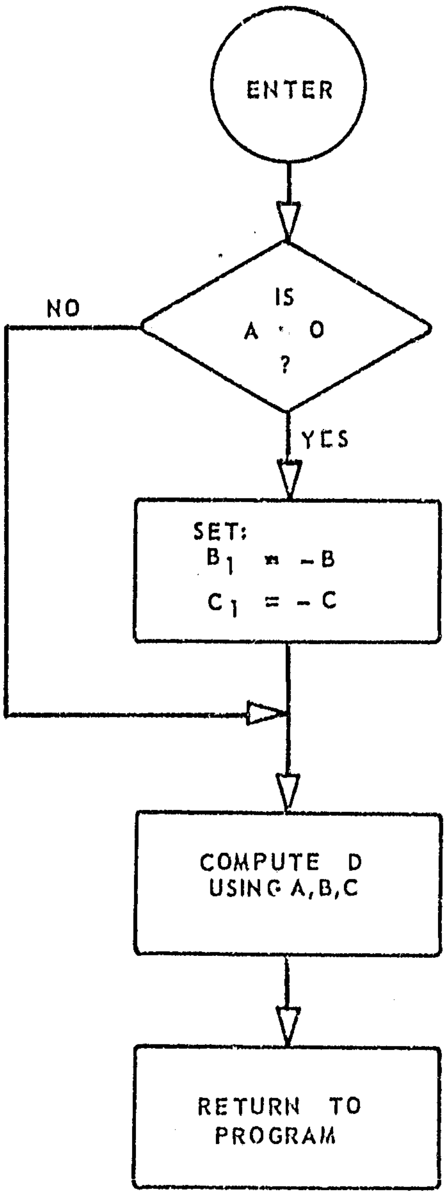
SECTION VI
COMPUTER PROGRAM FLOW CHART




PAGE 6-1
2
CALL DEGRAD TO CONVERT ϕ_J, AND λ_J TO RADIANS
CALL GEOXYZ TO CONVERT ϕ_J, λ_J, h_J, TO X_J, Y_J, Z_J
READ INPUT ACCURACIES σ_L, σ_M, σ_R, $\sigma_{\dot{L}}$, $\sigma_{\dot{M}}$, $\sigma_{\dot{R}}$
3
PAGE 6-1
PRINT INPUT ACCURACIES
COMPUTE BASELINE DIRECTION COSINES
READ START TIME = TSTART STOP TIME = TSTOP
5
PAGE 6-3
READ TIME X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$
4
4
IS TIME $\geq$ TSTART ?
NO
5
YES
IS TIME $\leq$ TSTOP
NO
YES
WRITE EOF
HALT (EXIT)
COMPUTE VEHICLE COORDINATES WITH RESPECT TO BASELINE CENTERS AND RANGE TRACKING POINT
CALL PHIGAM TO COMPUTE PARTIAL DERIVATIVE MATRIX $\overline{B}$
FORM POSITION COVARIANCE MATRIX $\overline{S}_1$ FROM σ_L, σ_M, σ_R
CALL VARCOV TO COMPUTE POSITION SIGMAS σ_X, σ_Y, σ_Z
FORM VELOCITY COVARIANCE MATRIX $\overline{S}_3$ FROM $\sigma_{\dot{L}}$, $\sigma_{\dot{M}}$, $\sigma_{\dot{R}}$
6
PAGE 6-3

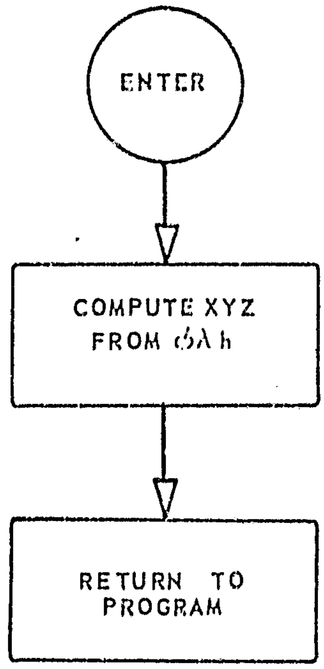

PAGE 6-2
6
CALL VARCOV TO COMPUTE VELOCITY SIGMAS $\sigma_{\dot{X}}, \sigma_{\dot{Y}}, \sigma_{\dot{Z}}$
CALL SIGVEL TO COMPUTE VECTOR VELOCITY SIGMA σ_V
7
LINE = LINE + 1
IS LINE > 14
NO
YES
SLEW ONE PAGE
SET LINE = 1
7
PRINT TIME $\sigma_X, \sigma_Y, \sigma_Z, \sigma_V, \sigma_{\dot{X}}, \sigma_{\dot{Y}}, \sigma_{\dot{Z}}$
WRITE TIME $\sigma_X, \sigma_Y, \sigma_Z, \sigma_{\dot{X}}, \sigma_{\dot{Y}}, \sigma_{\dot{Z}}, \sigma_V$
5
PAGE 6-2

DEGRAD ROUTINE

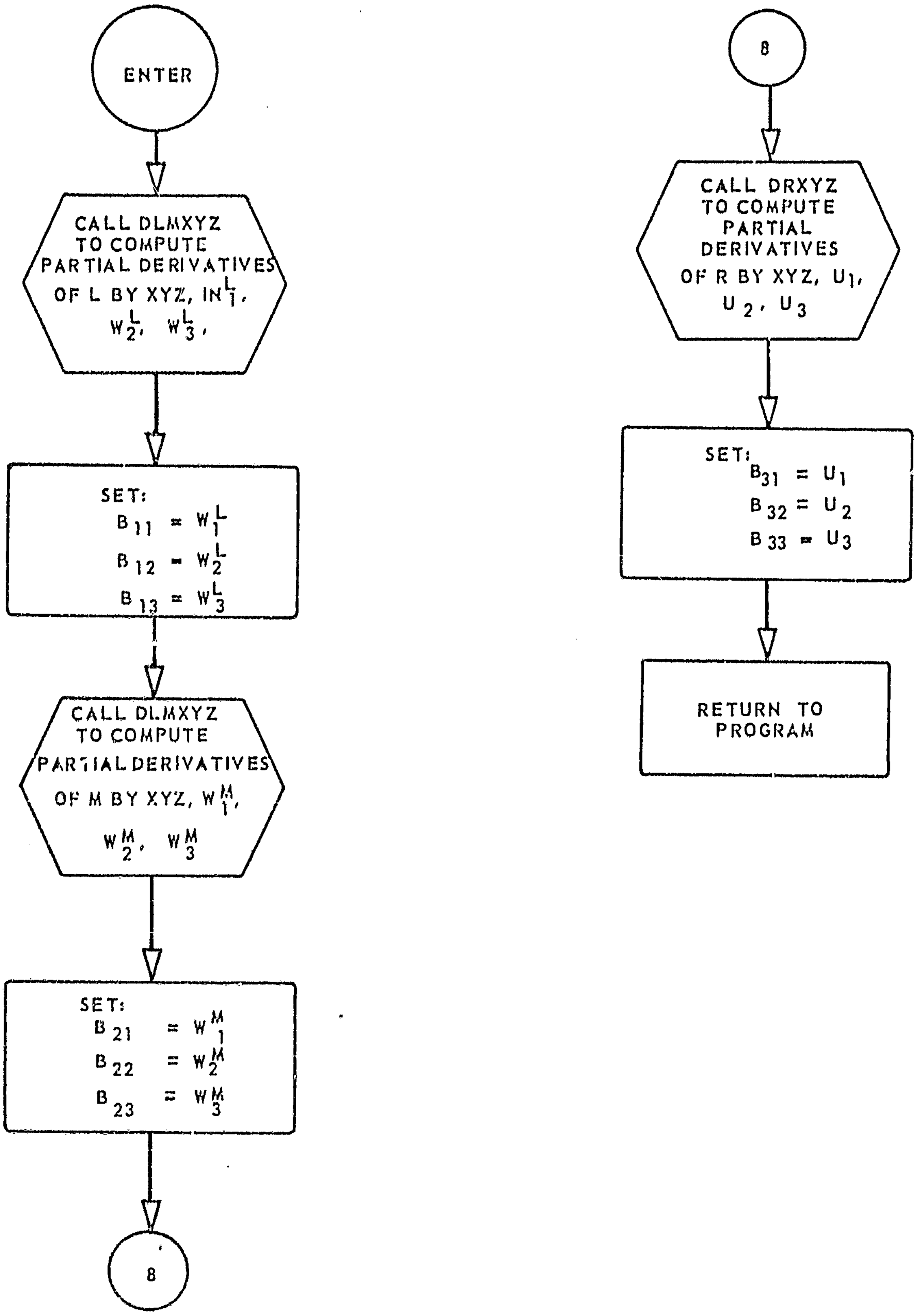


NOTE:
A = DEGREES
B = MINUTES
C = SECONDS
D = RADIANS

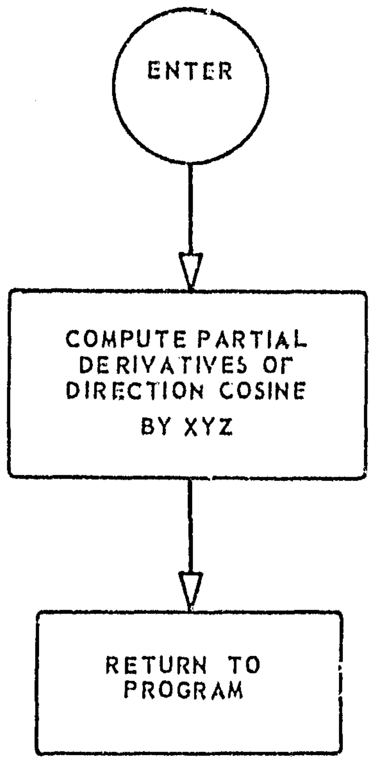
GEOXYZ ROUTINE



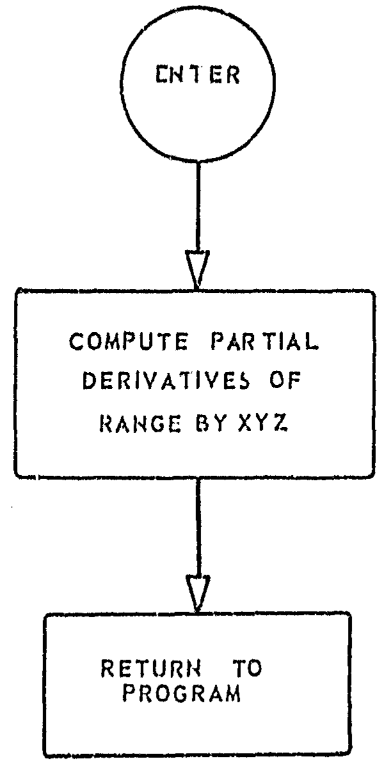
PHIGAM ROUTINE



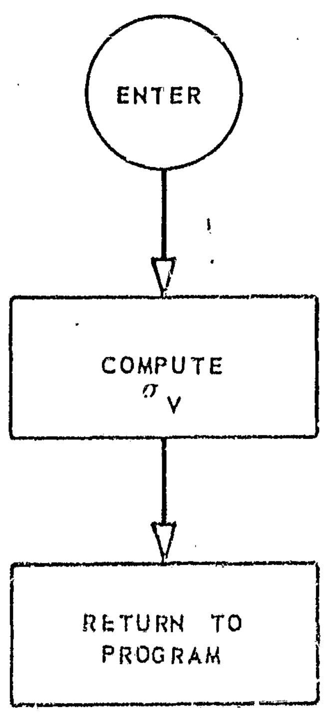
DLMXYZ ROUTINE



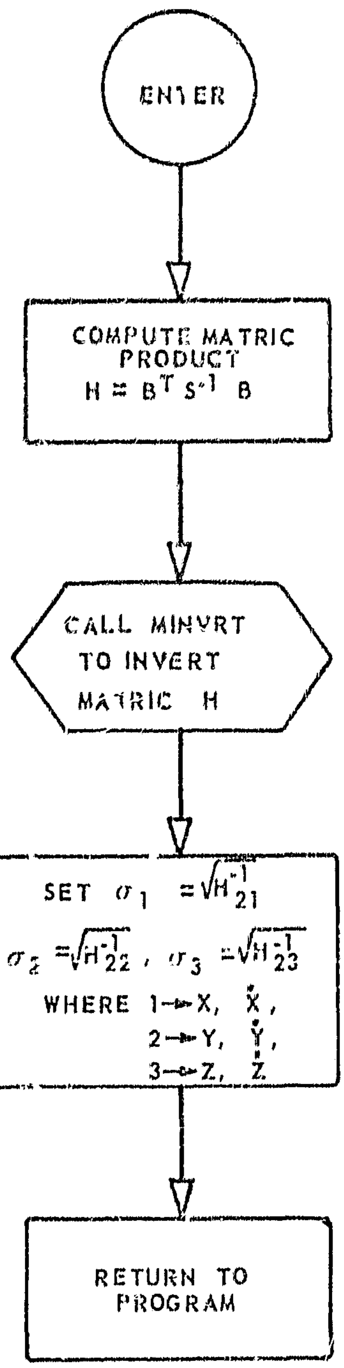
DRXYZ ROUTINE



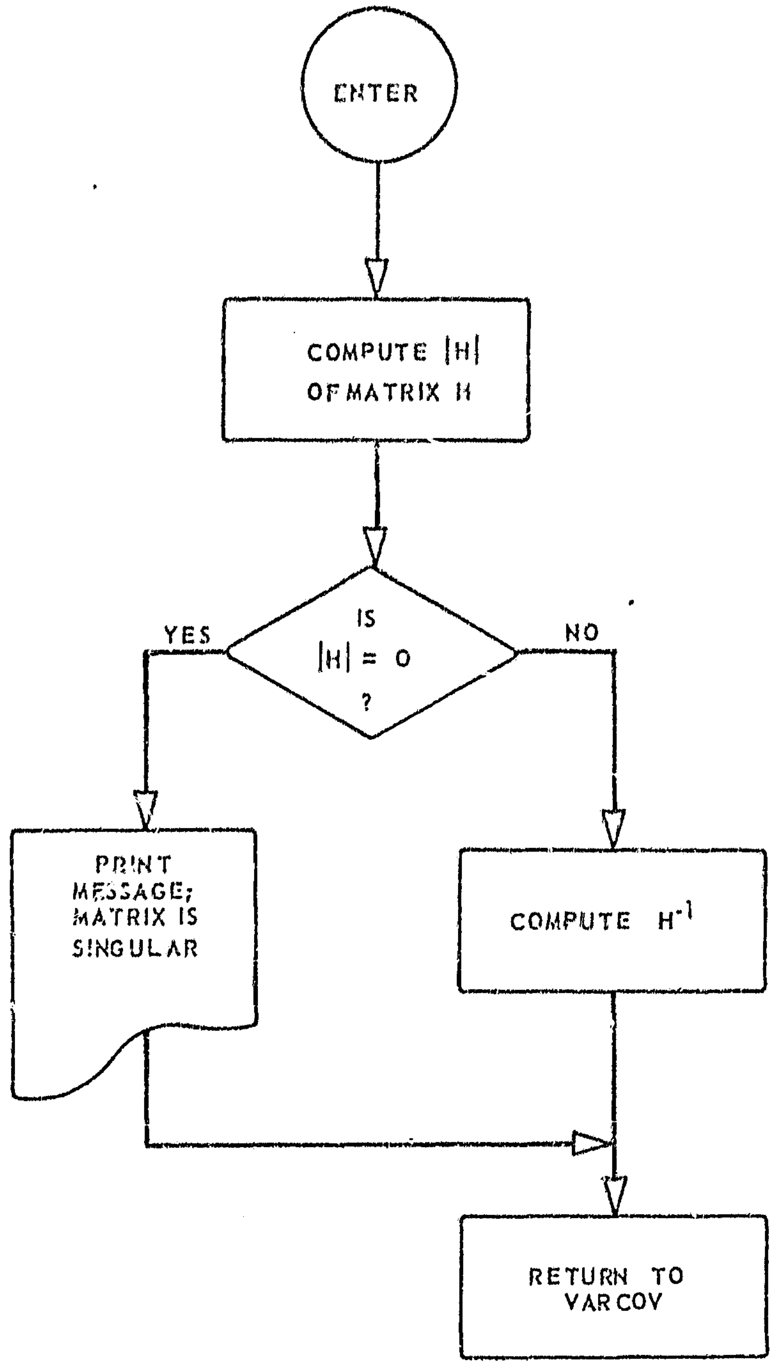
SIGVEL ROUTINE



VARCOV ROUTINE



MINVRT ROUTINE



SECTION VII
REFERENCES

1. Conversion of Geographic Coordinates to Cartesian Coordinates, NASA Document SP-57-E, by James H. Anderson.

2. A Treatment of Analytical Photogrammetry, RCA Data Reduction Technical Report No. 39, August 20, 1957, by Duane C. Brown.

APPROVAL

GP-832

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